CONTACT

- Mechanical interaction of bodies via surfaces
- Surfaces must “touch”
- Forces press bodies together
- Size (area) of contact dependent on forces, materials, geometry, temperature, etc.
- Contact mechanics

NATURE OF CONTACT

Rough surfaces contact

Highest peaks on highest peaks

Contact area = discrete islands

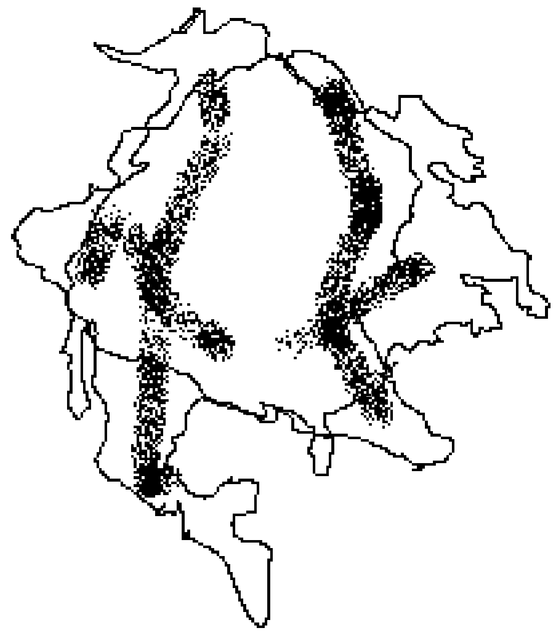
Real contact area small fraction of apparent area

Consequence

- contact stresses higher
- heating (friction) more intense
- electrical contacts: local constriction resistance

CONTINENTAL ANALOGY OF CONTACT

- Earth's surface rough: mountains & valleys



- Place South America on North America

- Contact: highest peaks against highest peaks
  - Andes/Appalachia
  - Highlands/Rockies

- Apparent contact area large, real contact area small

- **Contact between bodies similar**

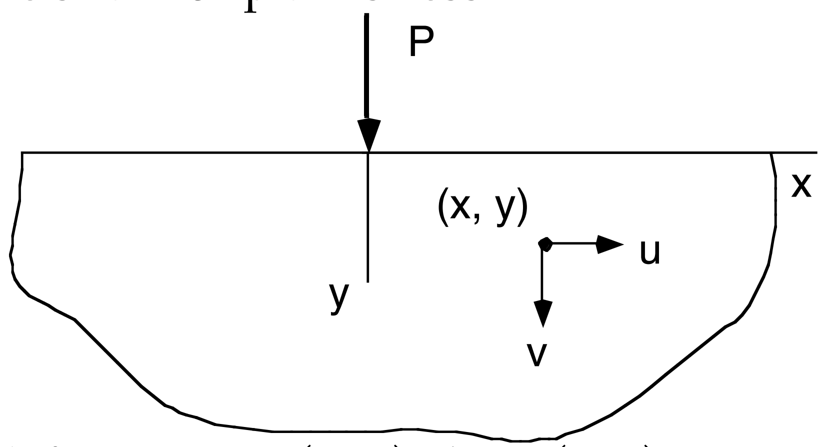
CONTACT MECHANICS FUNDAMENTALS

- Fundamental Solutions:
  - Boussinesq: 3D Elastic deformations from point force normal to semi-infinite space
  - Flamant: 2D Elastic deformations from point force normal to semi-infinite space

- Contact between spheres
  elastic: Hertzian contact
  plastic: Indentation (Meyer) hardness
  overall
  quirks

FLAMANT SOLUTION (2D)

- Point force P, normal to semi-infinite elastic space
- 2D: plane strain or plain stress



- Elastic deformations: (u, v) along (x, y)

$$u = -\frac{P}{4\pi\mu}\left\{(\kappa - 1)\theta - \frac{2xy}{r^2}\right\}, \quad v = -\frac{P}{4\pi\mu}\left\{(\kappa + 1)\log r - \frac{2y^2}{r^2}\right\}$$

- Stresses:

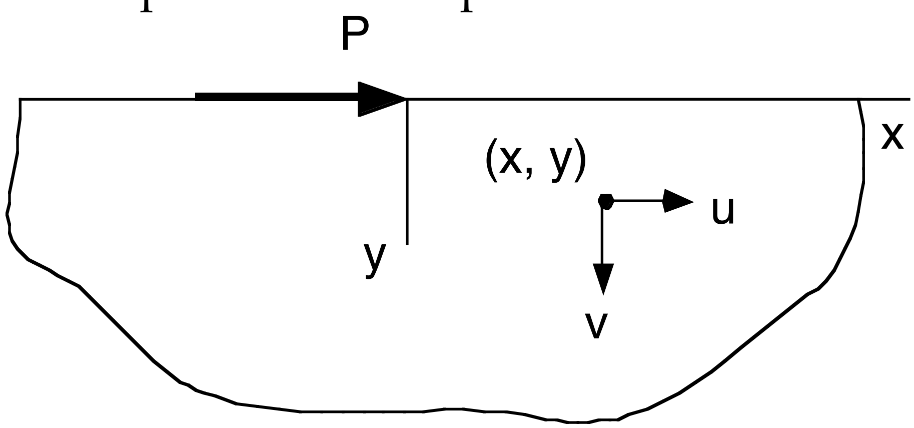
$$\sigma_{xx} = -\frac{2P}{\pi}\left\{\frac{y}{r^2} - \frac{y^3}{r^4}\right\}, \quad \sigma_{yy} = -\frac{2P}{\pi}\left\{\frac{y^3}{r^4}\right\}, \quad \sigma_{xy} = -\frac{2P}{\pi}\left\{\frac{xy^2}{r^4}\right\}$$

μ: elastic shear modulus, ν: Poisson's ratio
Elastic modulus: $E = 2(1 + \nu)\mu$
Dundars constant: $\kappa = 3 - 4\nu$, plane strain,
$= (3 - \nu)/(1 + \nu)$, plane stress

$$r = \sqrt{x^2 + y^2}, \quad \tan\theta = x/y$$

SOLUTION (2D)

- Point force P, tangential to semi-infinite elastic space
- 2D: plane strain or plain stress



- Elastic deformations: (u, v) along (x, y)

$$u = -\frac{P}{4\pi\mu}\left\{(\kappa+1)\log r + \frac{2y^2}{r^2}\right\},\quad v = \frac{P}{4\pi\mu}\left\{(\kappa-1)\theta + \frac{2xy}{r^2}\right\},$$

- Stresses:

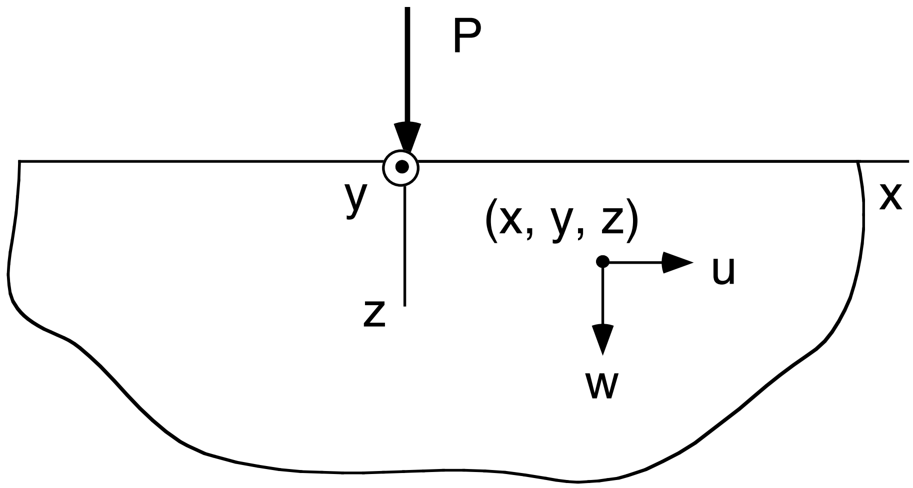
$$\sigma_{xx} = \frac{2P}{\pi}\left\{-\frac{x}{r^2} + \frac{xy^2}{r^4}\right\},\quad \sigma_{yy} = -\frac{2P}{\pi}\left\{\frac{xy^2}{r^4}\right\},\quad \sigma_{xy} = \frac{2P}{\pi}\left\{-\frac{y}{r^2} + \frac{y^3}{r^4}\right\}$$

μ: elastic shear modulus, ν: Poisson's ratio
κ: Dundars constant
$= 3 - 4\nu$, plane strain; $= (3 - \nu)/(1 + \nu)$, plane stress

$$r = \sqrt{x^2 + y^2},\quad \tan\theta = x/y$$

BOUSSINESQ SOLUTION (3D)

- Point force P on semi-infinite elastic space



- Elastic deformations: (u, v, w) along (x, y, z)

$$u = \frac{P}{4\pi\mu} \left\{ \frac{x\,z}{r^3} - (1 - 2\nu)\frac{x}{r\,(r + z)} \right\}$$

$$v = \frac{P}{4\pi\mu} \left\{ \frac{y\,z}{r^3} - (1 - 2\nu)\frac{y}{r\,(r + z)} \right\}$$

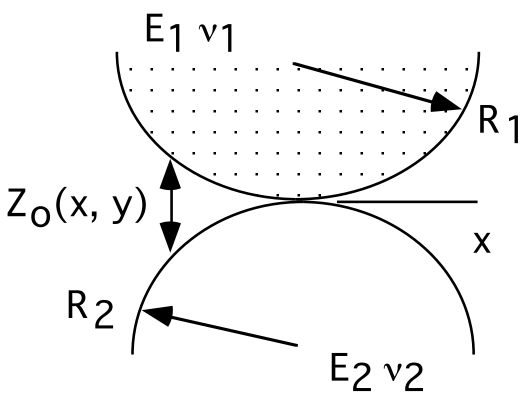
$$w = \frac{P}{4\pi\mu} \left\{ \frac{z^2}{r^3} + \frac{2(1 - \nu)}{r} \right\}$$

μ: elastic shear modulus, ν: Poisson's ratio

$$r = \sqrt{x^2 + y^2 + z^2}$$

HERTZIAN CONTACT THEORY

- Contact between elastic curved bodies
- Initial Contact



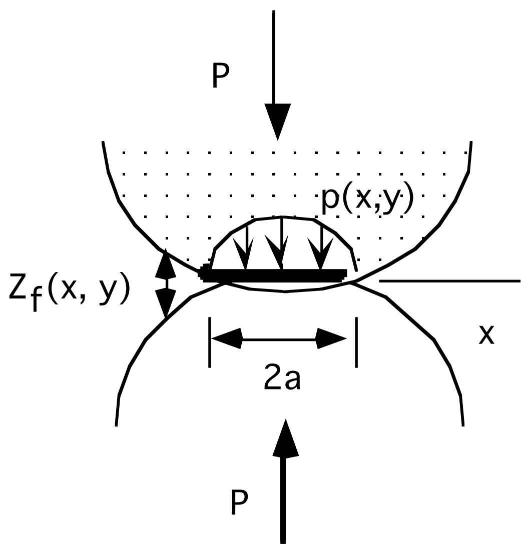
initial contact

- bodies (spheres) touch at origin
- curvatures R_1, R_2
- elastic parameters (E_1, ν_1) (E_2, ν_2)
- initial separation (parabolas)

$$Z_o(x, y) = \frac{x^2 + y^2}{2}\left\{\frac{1}{R_1} + \frac{1}{R_2}\right\}$$

HERTZIAN CONTACT THEORY

- Compressive normal load P
- Induces contact pressures p = p(x, y)



- normal surface deformations w_1, w_2
  $w_i = w_i[p(x, y); E_j, \nu_j, R_j]$
  (from Boussinesq)

- final separation

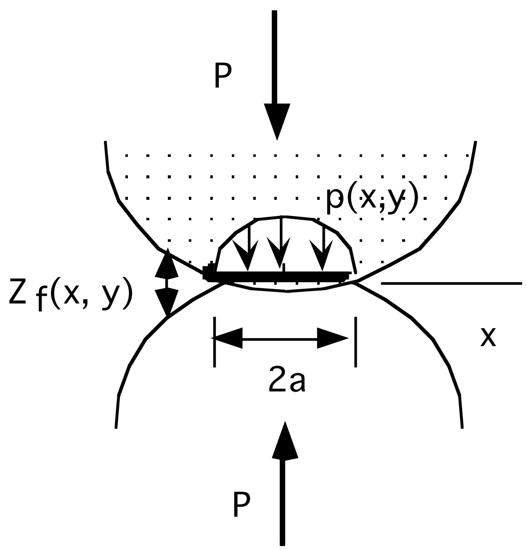
$$Z_f(x, y) = Z_o(x, y) - \alpha + w_1 + w_2$$

- *contact diameter 2a*, defines contact area

- *normal approach* α, amount centers of bodies come together

HERTZIAN CONTACT THEORY

Problem Statement



Unknowns

- a, α (eigenvalues)
- Contact pressures p(x, y) (eigenfunction)

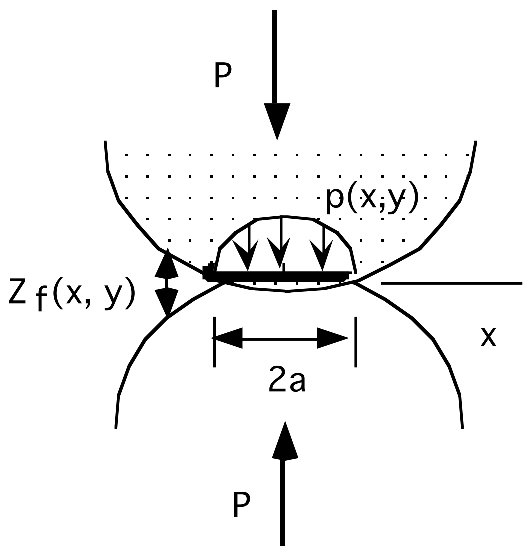
Physics: static equilibrium

Boundary Conditions

- over contact ($x^2 + y^2 < a^2$)
  $Z_f(x, y) = 0$, $p(x, y) \geq 0$
- outside ($x^2 + y^2 \geq a^2$)
  $p = 0$
- $P = \int_{\text{contact area}} p(x, y)\, dx\, dy$

HERTZIAN CONTACT THEORY

Solution



Sphere on Sphere (point contact):

- $p(x, y) = po\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{a^2}}$

$$po = \frac{3P}{2\pi a^2}$$

- $a = \left\{ \frac{3\pi P(k_1 + k_2)R_1 R_2}{4(R_1 + R_2)} \right\}^{\frac{1}{3}}$

$$k_i = \frac{1 - \nu_i^2}{\pi E_i}$$

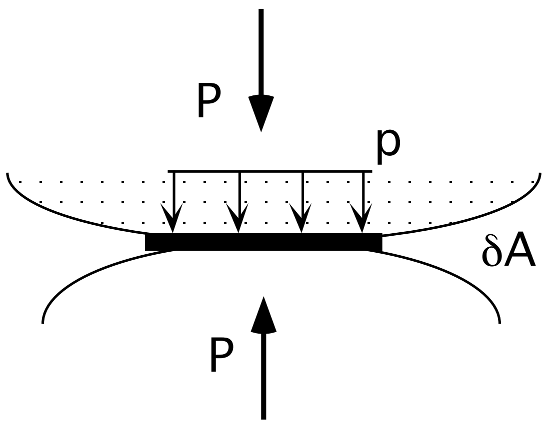
- $\alpha = \left\{ \frac{9\pi^2 P^2 (R_1 + R_2)(k_1 + k_2)^2}{16 R_1 R_2} \right\}^{\frac{1}{3}}$

Cylinder on Cylinder (line contact):

- $p(x) = p_o\sqrt{1-\frac{x^2}{a^2}}$, $p_o = \frac{2P}{\pi a l}$
- $a = \left[\frac{4P(k_1+k_2)R_1R_2}{l(R_1+R_2)}\right]^{1/2}$, $k_i = \frac{1-\nu_i^2}{\pi E_i}$
- $\alpha = \frac{P}{l}(k_1+k_2)\left[1+\ln\left\{\frac{4l^3}{(k_1+k_2)P}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\right\}\right]$
- l: length of contact along axis of cylinders

• PLASTIC CONTACT THEORY

- Indentation (Meyer) hardness



Bodies in contact

Load P > elastic limit
⇒ plastic deformations

- Contact pressures p(x, y) approximately uniform

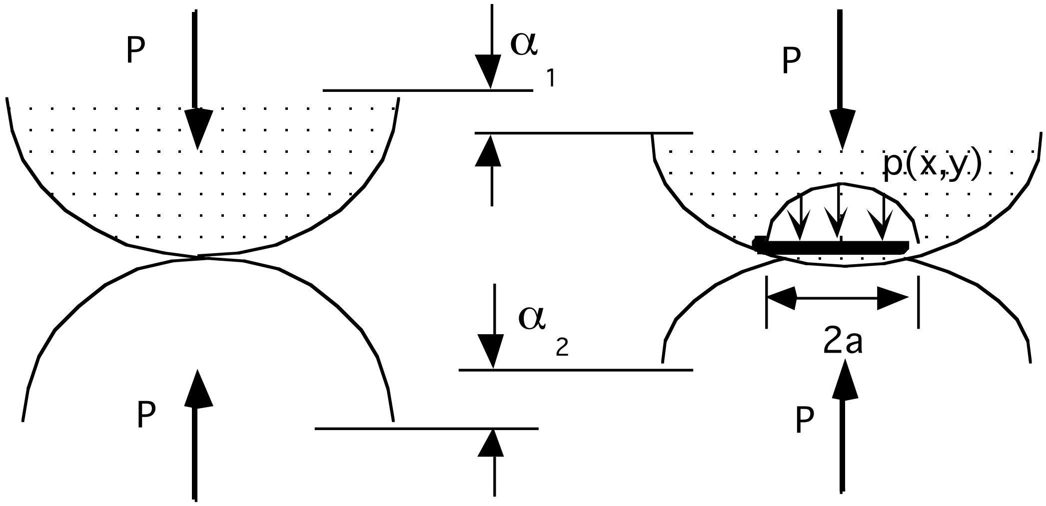
- Hardness pressure (indentation hardness)

$$H \equiv p \approx \frac{P}{\delta A}$$

H ≈ 3 x Yield stress

- Use: estimate contact area, given H and P

OVERALL CONTACT MODEL



- Spheres
- Increasing normal load P

$0 \leq P < P_e$; Elastic (Hertzian) contact model

$$\alpha = \alpha_1 + \alpha_2 = \left\{ \frac{9\pi^2 P^2 (k_1 + k_2)^2 (R_1 + R_2)}{16 R_1 R_2} \right\}^{\frac{1}{3}}$$

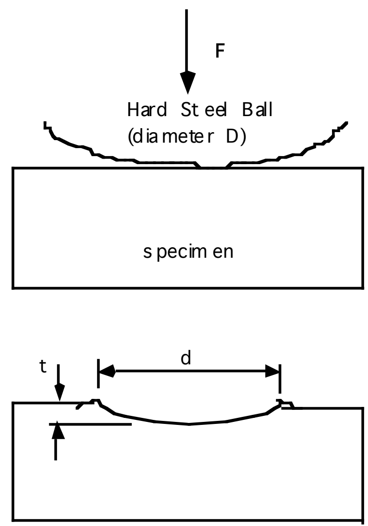
$P \geq P_e$; Elastic-Plastic contact model

$$\alpha = \alpha_1 + \alpha_2 > \left\{ \frac{9\pi^2 P_e^2 (k_1 + k_2)^2 (R_1 + R_2)}{16 R_1 R_2} \right\}^{\frac{1}{3}}$$

- Similar formulations, tangential loads & deformations

$P >> P_e$, Meyer Hardness problem

Hardness (Indentation) Test



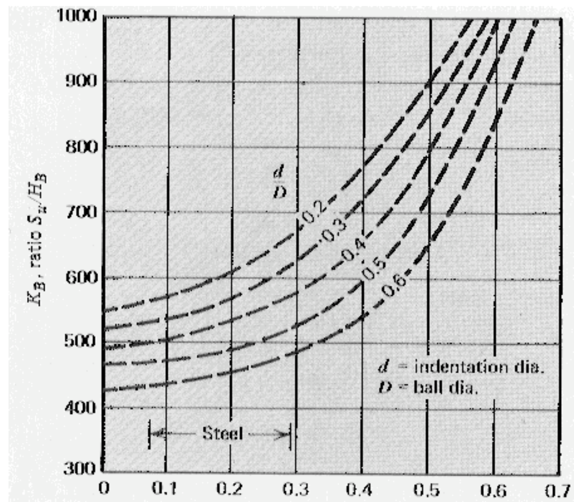
- Brinell hardness H_B
  - Hard steel ball (diameter D, load F)
  - indent for 30 sec.
  - measure permanent (plastic) set
  - $H_B = F/(\pi D t)$ [N/m^2] (units of stress)
  - $t \approx \{D - (D^2 - d^2)^{1/2}\}/2$
- Other hardness tests
  - Vickers: Diamond point indentation
  - Rockwell: measured like Brinell or Vickers
  - scratch test

Brinell Hardness

- For steels S = S(H)
- Ultimate strength

  $S_u = K_B H_B$
- Yield Strength

  $S_y \approx 1.05S_u - 30\text{ksi}$



Micro Hardness Measurements

Micro hardness tester:

- Indents specimen
- Use on thin films, e.g. hard drive coatings

CONTACT QUIRKS

- Nonlinear contact stiffness $P = P(\alpha)$

$$P = C\,\alpha^{3/2} \quad , C = \frac{4}{3\pi(k_1+k_2)}\sqrt{\frac{R_1R_2}{R_1+R_2}}$$

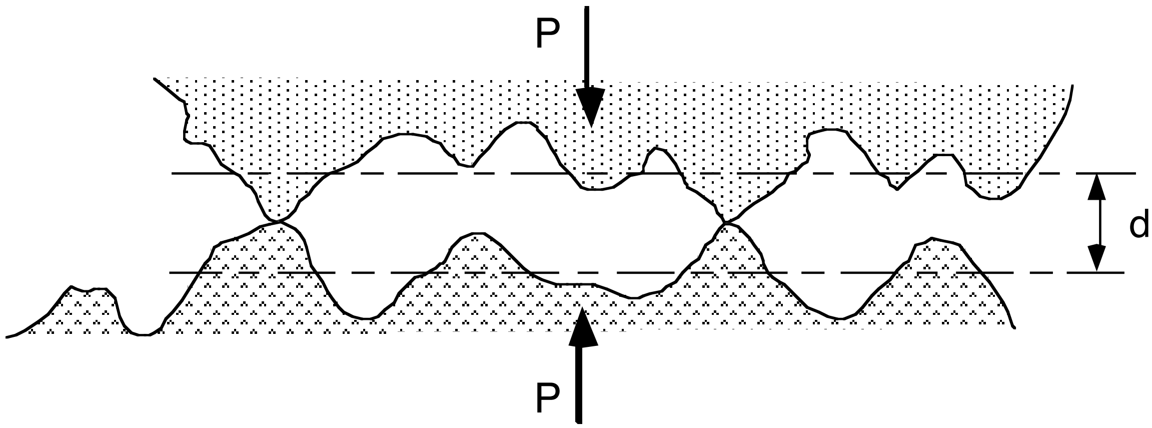
- $\Rightarrow$ nonlinear contact vibrations
- Tangential motions: slip/stick

  with friction

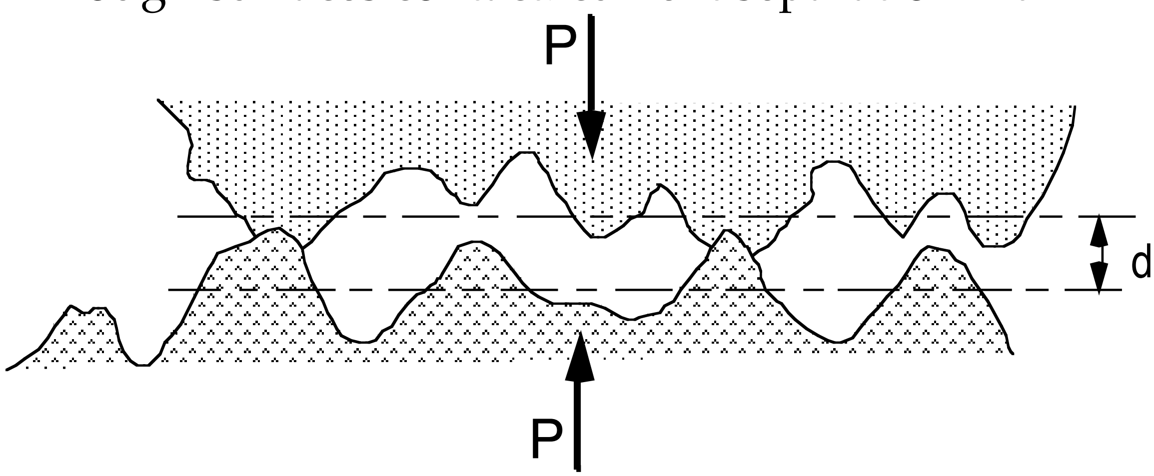
  high pitched "squeal" / fingernail on blackboard

ROUGH CONTACT MODELS

- Greenwood & Williamson



- Rough surfaces contact: current separation = d

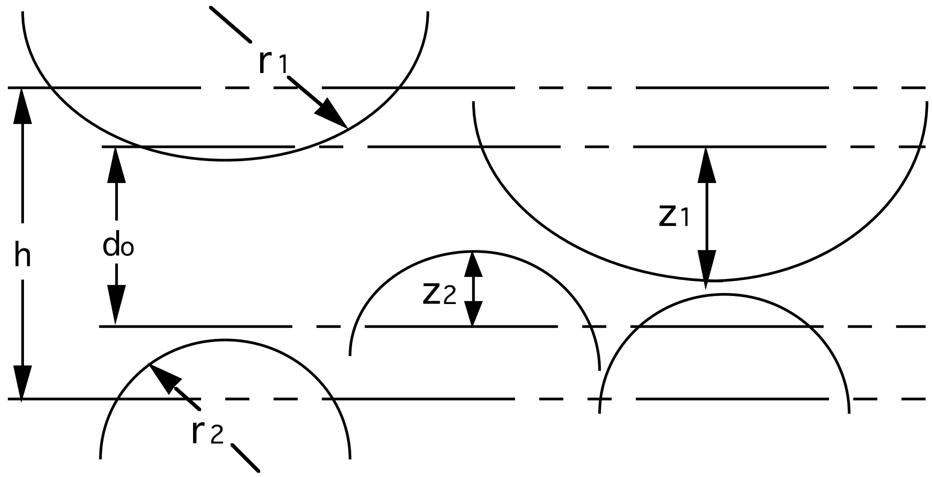


- Asperities contact

interference $\alpha = (z_1 + z_2) - d$

z_1 , z_2 surface heights, upper & lower

- Model asperities as spheres



- Relate contact quantities to surface heights $z = z_1 + z_2$ (random variable)

- Expected values ⇒ Macroscopic Contact Parameters

$$E[H(z)] = N \int_{d}^{\infty} H(z)F(z)dz$$

N: total number asperities
H(z): physical quantity, dependent on heights z
Lower limit: heights $z \geq d$ for asperities to touch

- Microscopic understanding ⇒ Important practical engineering parameters

- Contact force (elastic) on i<u>th</u> asperity

Asperity force: $P_i(z) = C\,\alpha^{3/2} = C\,(z - d)^{3/2}$ (Hertz)

Total force: $H(z) = P_i(z)$

$$P = E[P_i(z)] = N \int_d^\infty P_i(z) F(z)\, dz$$

- Real contact area

Asperity area: $H(z) = A_i(z) = \pi a^2$

From Hertz: $a = C_1\,\alpha^{1/2} = C_1\,(z - d)^{1/2}$

- Contact conductance

Asperity conductance: $G_i(z) = \rho/2a$ (Holm)